Factors: Homework

**Due: April 1st, 2020**

1.) Below are some common statements that elementary children believe. Are these statements true or false? Explain why each statement is true or provide a counterexample if it is false.

a) Even numbers have more factors than odd numbers.

b) Odd numbers have an odd number of factors.

c) Even numbers have an even number of factors.

d) The bigger a number is, the more factors it has.

e) After a while, the prime numbers stop. Really big numbers are all composite.

**Answer:** a.)

b.)

c.)

d.)

e.)

5.) A group of your students are exploring square numbers. They come up with these two conjectures:

* If you add two square numbers together, you get another square number. For example, 9+16=25.
* If you multiply two square numbers together, you get another square number. For example, 4 x 9 = 36

Discuss the validity of these two statements. What examples might you have your students explore to test these conjectures? How would you help them reason about whether these statements are true or false?

**Answer:**

7.) Your class is working on factoring large numbers. A group discussion about factoring the number 1000 is below.

Mei: “This is going to take forever! We have to try every number from 1 to 1000.”

Juan: “No, we only have to go to 500. Factors come in pairs, one large and one small. When we find the small factor, we’ll find the larger partner.”

Mei: “Still, 500 is a lot of numbers!”

a) Explain Juan’s idea. Is he correct?

b) Address Mei’s concern. Does she have to try every number from 1 to 500?

c) Do they have to go all the way to 500, or is there an earlier point at which they can stop?

d) What is the greatest number you have to test when finding all the factors of a number?

**Answer:** a.)

b.)

c.)

d.)

8.) Which of the following numbers have an odd number of factors? Explain each choice.

i. 5^2 x 3^2 ii. 4^6 iii. 3^4 x 11^2 x 13

**Answer:**

Prime Factorization: Homework

**Due: April 1st, 2020**

1.) Consider the number N = 16^4 × 81^4 × 6^6.

a) Find the prime factorization of N.

b) Provide two other factorizations of N.

c) Without multiplying, briefly explain how you know that the two factorizations you provided in part b are factorizations of N.

**Answer:** a.)

b.)

c.)

2.) Consider the number M = 7^2 × 23 × 31, where 7, 23, and 31 are prime numbers.

a) List all of the factors of M without computing the value of M first. Clearly show or explain how you were able to find all of the factors.

b) Does the number K = 2 × 52 × 13 have the same number of factors as M? How can you tell without finding all the factors of K?

**Answer:** a.)

b.)

3.) If both 3 and 5 are factors of a number x, must 15 be a factor of x? Why or why not?

**Answer:**

4.) What is the Fundamental Theorem of Arithmetic and why is it important?

**Answer:**

5.) Two positive integer numbers are called relatively prime or coprime if they have no common factors other than 1.

a) List three pairs of numbers which are relatively prime.

b) List three pairs of numbers which are not relatively prime.

**Answer:**

Divisibility: Homework

**Due: April 1st, 2020**

1.) Determine whether the number N = 5^3 x 2 x 3^2 x 19, written in prime factored form, has each of the following divisors without finding N. Briefly explain how you know.

a) 3

b) 14

c) 15

d) 30

**Answer:** a.)

b.)

c.)

d.)

2.) Is 7^3 x 3^4 a divisor of 7^3 x 3^2? Explain why or why not.

**Answer:**

3.) Consider the number M = 31 × 7^2 × 23, where 31, 7, and 23 are prime numbers. List all of the divisors of M without computing the value of M first. Clearly show or explain how you were able to find them all.

**Answer:**

4.) A divisibility rule or test is a quick way to check if a counting number is divisible by a given number without actually dividing.

a) Apply the divisibility test for 3 to 125 and 415.

b) Apply the divisibility test for 9 to 259 and 3,456.

c) Apply the divisibility test for 4 to 3894 and 972.

**Answer:**